JOURNAL OF SCIENCE TECHNOLOGY AND EDUCATION 14(1), MARCH, 2026
E-ISSN: 3093-0898, PRINT ISSN: 2277-0011; Journal homepage: www.atbufstejoste.com

Constructing Algebraic Structures using the Elements of Real Valued Type 1 Level 1 Fuzzy Sets

Abubakar I.
Department of Mathematics
Federal University of Education Zaria

ABSTRACT
Algebraic Structures such as Groupoids, Semigroups, Monoids, Groups, Rings and Fields, are usually constructed using the elements of classical set. However, in this paper we aimed at constructing such structures using the elements of real valued type 1 level 1 fuzzy sets. We applied the addition and multiplication operations on the element of real valued type 1 level 1 fuzzy sets with the aim of detecting the existence of properties such as closure, Commutativity, associativity, identity elements and inverses of elements and we derived conclusions based on the observations made through the use of illustrative examples.

ARTICLE INFO
Article History
Received: November, 2025
Received in revised form: December, 2026
Accepted: February, 2026
Published online: March, 2026

KEYWORDS
Algebraic Structures, classical set, type 1 level 1 fuzzy sets, Semigroups, Groups

INTRODUCTION

A Fuzzy set is a collection of objects with various degrees of membership with respect to the collection. Each element in the set is expressed as an ordered pair comprising of the element and its membership grade (or membership degree or membership function). The membership grade in each case takes value within the interval [0, 1]. To construct algebraic structures such as Semigroup and groups, binary operations such as addition and multiplication are required. In fuzzy set theory, these operations are performed using special fuzzy arithmetic operations using max-min logical operators.

PRELIMINARIES

Real valued Fuzzy Set:

This is a fuzzy set of the form $\bar{A}$ $=\{x_{\mathrm{i}}, m_{\bar{A}}(x_{\mathrm{i}}) \,/\, x_{\mathrm{i}} \in \mathrm{X}\}$, i = 1, 2, 3,,.. n, where, $X \in$ IR, $\boldsymbol{m_{\bar{A}}(x_{\mathrm{i}})} \in$ **[0, 1]** (Each $m_{\bar{A}}(x_{\mathrm{i}})$ is the membership grade of an element x_{i} in $\bar{A}$). Each element is of the form **$\bar{a}$ = (x, $m_{\bar{A}}(x)$),** where x is a real number.

Type of a Fuzzy Set:

The type of a fuzzy set depends on the nature of the membership grades of its elements. For example, in a type 1 fuzzy set, the membership grades of its elements are real numbers within [0, 1]. For a type2 fuzzy set, the membership grades of its elements are type 1 fuzzy sets.

Level of a Fuzzy Set:

The level of a Fuzzy Set depend on the nature of the elements that are attached to the membership grades in the set. For example, in a real valued level 1 fuzzy set, the attached elements are real numbers while in a level 2 fuzzy set. the attached elements are type 1 fuzzy sets.

Type 1 Level 1 Real valued Fuzzy Set:

This is a Fuzzy Set of the form: $\bar{A}$ = $\{x_{\mathrm{i}}, m_{\bar{A}}(x_{\mathrm{i}}) \,/\, x_{\mathrm{i}} \in \mathrm{X}\}$, i = 1, 2, 3... n, where, $X \in$ IR, $m_{\bar{A}}(x_{\mathrm{i}}) \in$ [0, 1]. Example 1.1 $\bar{A}$ = $\{(3, 0.5), (7, 0.9), (6, 0.3)\}$, $\bar{A}$ is a type1level 1 fuzzy set consisting of three elements

Arithmetic Operations in real valued type 1 level 1 Fuzzy Sets.

Given two elements $\bar{a}$, $\bar{b}$ in the set of real valued type 1 level 1 fuzzy sets $\bar{A}$ such that, $\bar{a}$ = (x, m_1) $\bar{b}$ = (y, m_2) where, $x, y \in$ IR, $m_1, m_2 \in$ [0, 1]

$\bar{a} + \bar{b} = (x + y, \min(m_1, m_2))$

$\bar{a} - \bar{b} = (x - y, \min(m_1, m_2))$

Corresponding author: Abubakar I.
ibrabu261@gmail.com
Department of Mathematics, Federal University of Education Zaria.

JOURNAL OF SCIENCE TECHNOLOGY AND EDUCATION 14(1), MARCH, 2026
E-ISSN: 3093-0898, PRINT ISSN: 2277-0011; Journal homepage: www.atbufstejoste.com

$\bar{a}.\bar{b}$ = $(x.y, \min(m_1, m_2))$
$\bar{a}$ / $\bar{b}$ = $(x/y, \min(m_1, m_2))$

METHODOLOGY

We present the applications of fuzzy addition and multiplication operations on the elements of some samples real valued type 1 level 1 real valued fuzzy sets with aim of detecting the existence of Semigroup and Group properties such as closure, associativity, identity element and inverses of elements, possibility of Commutativity among elements shall also be examined. The addition and multiplication operations were carried out using the fuzzy addition and multiplication operations characterised by the use of max-min operators in contrast with the algebraic sum and algebraic product respectively. All conclusions shall be made based on the outcomes derived from the examples in the format; Example, Observation, Lemma / Theorem.

Illustrative Example

Let $\bar{a}, \bar{b}, \bar{c}$ be three elements in the set of real valued type 1 level 1 fuzzy sets $\bar{A}$ such that,
$\bar{a}$ **=** $(7, 0.4)$ $\bar{b}$ **=** $(3, 0.6)$ $\bar{c}$ **=** (5, 0.2)

Applying the addition operation

$\bar{a}$ **+** $\bar{b}$ = $(7 + 3, \min(0.4, 0.6))$ = (10, 0.4)
$\bar{a}$ **+** $\bar{b}$ is also a real valued type 1 level1 fuzzy set.
(closure property with respect to addition operation satisfied)

$\bar{b}$ **+** $\bar{a}$ = $(3 + 7, \min(0.6, 0.4))$
= $(10, 0.4)$ = $\bar{a}$ **+** $\bar{b}$
(Commutativity property with respect to addition operation satisfied)

($\bar{a}$ **+** $\bar{b}$)+ $\bar{c}$= $(7 + 3, \min(0.4, 0.6))$ +$(5, 0.2)$
= ((7 + 3) +5, min ((0.4, 0.6),0.2)) = (15, 0.2)
$\bar{a}$+ ($\bar{b}$ **+** $\bar{c}$) = (7, 0.4) +((3, 0.6) + (5, 0.2))
= ((7 + (3 +5), min ((0.4, (0.6, 0.2)) = (15, 0.2). Therefore, ($\bar{a}$ **+** $\bar{b}$)+ $\bar{c}$ = $\bar{a}$+ ($\bar{b}$ **+** $\bar{c}$)
(Associativity property with respect to addition operation satisfied)
Let I_0 = (0, 1)

$\bar{a}$+ I_0 = $(7, 0.4)$ + (0. 1) = (7+ 0, min (0.4, 1)) = (7, 0.4) = $\bar{a}$. Similarly, I_0 + $\bar{a}$ = (0. 1) +$(7, 0.4)$ = (0+ 7, min (1 0.4,)) = (7, 0.4) = $\bar{a}$. Therefore $\bar{a}$+I_0 = I_0+ $\bar{a}$ = $\bar{a}$ I_0 = (0, 1) is the fuzzy identity element with respect to addition operation in the set of real valued type 1 level1 fuzzy sets.

Let $\bar{a}^{-1}$ be the inverse of $\bar{a}$ such that $\bar{a}$+$\bar{a}^{-1}$ =I_0
$\bar{a}^{-1}$ = I_0 - $\bar{a}$= (0, 1) - (7, 0.4) = (0- 7, min(1, 0.4))
= (-7, 0.4)

Remark 1

Based on the above observations, Set of real valued type1 level 1 fuzzy sets satisfies all the properties of an abelian group with respect to addition operation.

Applying multiplication operation

$\bar{a}.\bar{b}$ = $(7(3), \min(0.4, 0.6))$ = (21, 0.4)
$\bar{a}.\bar{b}$ is also a real valued type1 level1 fuzzy set
(Closure property with respect to multiplication operation satisfied)

$\bar{b}.\bar{a}$ = (3, 0.6). (7, 0.4) = $(3(7), \min(0.6, 0.4))$ = (21, 0.4) = $\bar{a}.\bar{b}$
(Commutativity property with respect to multiplication operation satisfied)

($\bar{a}.\bar{b}$).$\bar{c}$ = $(21, 0.4)$.(5, 0.2) = (21(5), min (0.4, 0.2)) = (105, 0.2)
$\bar{b}.\bar{c}$ = (3, 0.6) (5, 0.2) = (3(5), min (0.6, 0.2)) = (15, 0.2)
$\bar{a}$.($\bar{b}.\bar{c}$) = (7, 0.4) (15, 0.2) = (7(15), min (0.4, 0.2)) = (105, 0.2). Therefore ($\bar{a}.\bar{b}$).$\bar{c}$ =$\bar{a}$. ($\bar{b}.\bar{c}$)
(Associativity property with respect to multiplication operation satisfied)

Let I_1 = (1, 1)
$\bar{a}.I_1$ =(x, m_1). (1. 1) = (x.1, min (m_1 , 1)) = (x, m_1) = $\bar{a}$. Similarly, I_1 . $\bar{a}$= (1. 1).(x, m_1) = (1. x, min (1 m_1 ,)) = (x, m_1) = $\bar{a}$
Therefore $\bar{a}.I_1$ = I_1 . $\bar{a}$ = $\bar{a}$
I_1 = (1, 1) is the fuzzy identity element with respect to multiplication operation in the set of real valued type 1 level1 fuzzy sets.

Let $\bar{a}^{-1}$ be the inverse of $\bar{a}$ such that $\bar{a}.\bar{a}^{-1}$ = I_1

Corresponding author: Abubakar I.
ibrabu261@gmail.com
Department of Mathematics, Federal University of Education Zaria.

JOURNAL OF SCIENCE TECHNOLOGY AND EDUCATION 14(1), MARCH, 2026
E-ISSN: 3093-0898, PRINT ISSN: 2277-0011; Journal homepage: www.atbufstejoste.com

$\bar{a}^{-1}$ = $^{I_1}/_{\bar{a}}$ = (1, 1)/(7, 0.4) = ($^1/_7$, min(1, 0.4)) = ($^1/_7$, 0.4)

This shows that the inverse of $\bar{a}$ = (7, 0.4) is ($^1/_7$, 0.4)

Remark 2

If $\bar{a}$ = (0, 0.4), $\bar{a}^{-1}$ = ($^1/_0$, 0.4), since $^1/_0$ is not defined, $\bar{a}^{-1}$ does not exists in this case, which means only non-zero elements have inverses in the set of real valued type1 level 1 fuzzy sets. Therefore, inverses of elements in the set of real valued type 1 level 1 fuzzy sets does not exists at some point in IR. Therefore, set of real valued type1 level 1 fuzzy sets satisfies only the properties of a commutative monoid with respect to multiplication operation.

To confirm Distributivity of multiplication operation over addition operation

$\bar{a}.(\bar{b} + \bar{c})$ = (7, 0.4) $(3 + 5, \min(0.6, 0.2))$
=$(7(3 + 5), \min(0.4, (0.6, 0.2)))$
=$(7(3) + 7(5)), \min((0.4, 0.6), (0.4, 0.2))$
= $(7(3), \min((0.4, 0.6)) + (7(5), (0.4, 0.2))$
= $\bar{a}.\bar{b} + \bar{a}.\bar{c}$

This shows that the multiplication operation is distributive over addition operation on the elements of real valued type 1 level 1 fuzzy sets.

RESULTS AND DISCUSSIONS

Set of elements of real valued type 1 level 1 fuzzy sets satisfies the following properties

(i) It forms an Abelian group with respect to addition operation
(ii) It is a commutative monoid with respect to multiplication operation
(iii) Its multiplication operation is distributive over its addition operation
(iv) Its additive and multiplicative identity elements are distinct.

Therefore, Set of real valued type 1 level 1 fuzzy sets satisfies all the properties of a commutative Ring with unity

Theorem:1.1

Set of real valued type 1 level 1 fuzzy sets forms a commutative Ring with unity

Proof:

Let $\bar{a}, \bar{b}, \bar{c}$ be three elements in the set of type 1 level1 fuzzy sets $\bar{A}$ such that,
$\bar{a} = (x, m_1)$ $\bar{b} = (y, m_2)$ $\bar{c} = (z, m_3)$. where, x, y, z ∈ IR, m_1, m_2, m_3 ∈ [0, 1].

We want to show that:

(a)With respect to addition operation

(i) $\bar{a} + \bar{b} \in \bar{A}$ (ii) $\bar{a} + \bar{b} = \bar{b} + \bar{a}$ (iii)$(\bar{a} + \bar{b}) + \bar{c} = \bar{a} + (\bar{b} + \bar{c})$. (iv) There exist $I_0 \in \bar{A}$ such that $\bar{a} + I_0 = I_0 + \bar{a} = \bar{a}$. ($I_0$ is the identity element with respect to addition operation). (v)For every element $\bar{a} \in \bar{A}$ there exist $\bar{a}^{-1} \in \bar{A}$ such that $\bar{a} + \bar{a}^{-1} = I_0$

(b) With respect to multiplication operation

(i) $\bar{a}.\bar{b} \in \bar{A}$ (ii) $\bar{a}.\bar{b} = \bar{b}.\bar{a}$ (iii) $(\bar{a}\,\bar{b}).\bar{c} = \bar{a}.(\bar{b}.\bar{c})$. (iv) There exist $I_1 \in \bar{A}$ such that $\bar{a}.I_1 = I_1.\bar{a} = \bar{a}$. ($I_1$ is the identity element with respect to multiplication operation). (v)For every element $\bar{a} \in \bar{A}$ there exist $\bar{a}^{-1} \in \bar{A}$ such that $\bar{a}.\bar{a}^{-1} = I_1$

Applying the addition operation

$\bar{a} + \bar{b} = (x + y, \min(m_1, m_2))$
$x + y$ ∈ IR, since x and y ∈ IR and
$\min(m_1, m_2)$ ∈ [0, 1] since m_1, m_2, ∈ [0, 1]
Therefore $\bar{a} + \bar{b} \in \bar{A}$ (closure property satisfied)

$\bar{b} + \bar{a} = (y + x, \min(m_2, m_1))$
= $(x + y, \min(m_1, m_2)) = \bar{a} + \bar{b}$
(Commutativity property satisfied)

$(\bar{a} + \bar{b}) + \bar{c} = (x + y, \min(m_1, m_2)) + (z, m_3)$
= (($x + y$) +z, min ((m_1, m_2), m_3))
= (x+(y+ z), min (m_1 ,(m_2, m_3)))
=$\bar{a} + (\bar{b} + \bar{c})$
(Associativity property satisfied)

Let I= (0, 1)

$\bar{a}$+I = (x, m_1) + (0. 1) = (x+ 0, min (m_1 , 1)) = (x, m_1) = $\bar{a}$

Similarly, I + $\bar{a}$= (0. 1) +(x, m_1) = (0+ x, min (1 m_1 ,)) = (x, m_1) = $\bar{a}$

Therefore $\bar{a}$+I = I + $\bar{a}$=$\bar{a}$

Corresponding author: Abubakar I.
ibrabu261@gmail.com
Department of Mathematics, Federal University of Education Zaria.

JOURNAL OF SCIENCE TECHNOLOGY AND EDUCATION 14(1), MARCH, 2026
E-ISSN: 3093-0898, PRINT ISSN: 2277-0011; Journal homepage: www.atbufstejoste.com

I= (0, 1) is the fuzzy identity element with respect to addition operation in the set of real valued type 1 level1 fuzzy sets.

Let $\bar{a}^{-1}$ be the inverse of $\bar{a}$ such that $\bar{a}+\bar{a}^{-1}$=I

$\bar{a}^{-1}$ = I - $\bar{a}$= (0, 1) - (x, m_1) = (0- x, min (1, m_1))

= (-x, min (1, m_1))

Therefore, set of real valued type1 level1 fuzzy sets satisfies all the properties of an abelian group with respect to addition operation

Applying multiplication operation

$\bar{a}$. b̄ = $(x.y, \min(m_1, m_2))$

$x.y$ ∈ IR since x and y ∈ IR and

$\min(m_1, m_2)$ ∈ [0, 1] since m_1 , m_2 ∈ [0, 1]

Therefore $\bar{a}$. b̄ ∈ $\bar{A}$ (Closure property satisfied)

$\bar{b}$. ā = $(y.x, \min(m_2, m_1))$

= $(x.y, \min(m_1, m_2))$ = $\bar{a}$. b̄

(Commutativity property satisfied)

($\bar{a}$. b̄). $\bar{c}$ = $(x.y, \min(m_1, m_2)).(z, m_3)$

= (($x.y$) +z, min ((m_1 , m_2), m_3))

= (x. (y. z), min (m_1 ,(m_2, m_3)))

=$\bar{a}$. ($\bar{b}$. c̄)

(Associativity property satisfied)

Let I= (1, 1)

$\bar{a}$.I =(x, m_1). (1. 1) = (x.1, min (m_1 , 1))= (x, m_1) = $\bar{a}$. Similarly, I. $\bar{a}$= (1. 1).(x, m_1) = (1. x, min (1 m_1 ,))= (x, m_1) = $\bar{a}$

Therefore $\bar{a}$.I = I. $\bar{a}$=$\bar{a}$

I= (1, 1) is the fuzzy identity element with respect to multiplication operation in the set of real valued type 1 level1 fuzzy sets.

Let $\bar{a}^{-1}$ be the inverse of $\bar{a}$ such that $\bar{a}.\bar{a}^{-1}$=I

$\bar{a}^{-1}$ = I / $\bar{a}$ = (1, 1)/(x, m_1)

= (1/x, min (1, m_1))

= (1/x, min (1, m_1))

We observed that at x=0, $\frac{1}{x}$ is not defined, therefore $\bar{a}^{-1}$ does not exists which means some elements of the set of real valued type1 level1 fuzzy sets have no inverses at a certain point in IR. Therefore, inverses of elements in the set of type 1 level 1 fuzzy sets does not generally exists. Therefore, set of real valued type1 level 1 fuzzy sets satisfies only the properties of a commutative monoid with respect to multiplication operation.

To confirm Distributivity of multiplication operation over addition operation

$\bar{a}$. ($\bar{b}$ + c̄) = (x, m_1) $(y + z, \min(m_2, m_3))$

=$(x(y + z), \min(m_1, (m_2, m_3)))$

=$(x.y + x.z), \min((m_1, m_2), (m_1, m_3)))$

= (x.y, $\min((m_1, m_2)) + (x.z, (m_1, m_3))$

=$\bar{a}$. $\bar{b}$ + $\bar{a}$. c̄

This shows that the multiplication operation is distributive over addition operation on the elements of real valued type 1 level 1 fuzzy sets.

CONCLUSION

We have seen that algebraic structures such as Semigroups, Monoids, Abelian groups and Rings can be constructed using the elements of real valued type 1 level 1 fuzzy sets.

REFERENCES

[1] Abubakar I. (2023) Constructing Algebraic structures of type 1fuzzy sets, *M.SC Dissertation* Mathematics Department Ahmadu Bello University, Zaria Nigeria

[2] Alkali A.J., (2015), A Study of Fuzzy Multisets and their algebra, *Ph.D. Thesis* Mathematics Department Ahmadu Bello University Zaria, Nigeria

[3] Atanasov K (1986), Intuitionistic fuzzy sets theory and applications *Physica vertag, Hei delberg Germany*.4; 17-19

[4] Atanasov K, (2016), Similarity measures for intuitionistic fuzzy set. *International workshop on computational intelligence*, 4; 5-9

[5] Atanasov K (2017), Type 1 Fuzzy sets and intuitionistic fuzzy sets. *Fuzzy sets and systems*. 20(1); 87-96.

[6] Chakraberty S., (2016) Fuzzy logic and set theory. *Unpublished YouTube Lecture series*, 1;1- 22

[7] Debashree G., Debjani G. (2010), A new approach to distance measure and similarity measure. *Journal of mathematical analysis and applications* 10 (1), 90 – 99.

[8] Goguen, J. A., (1967), L. Fuzzy sets, *Journal of mathematical analysis and applications*, 18; 145 – 174

Corresponding author: Abubakar I.
ibrabu261@gmail.com
Department of Mathematics, Federal University of Education Zaria.

[9] Guha D., Chakraberty D., (2010) Fuzzy sets and fuzzy relations. *Journal of Mathematical analysis and applications*, 61; 399 – 421

[10] Hussain M., (2010), Fuzzy relations M.Sc. Thesis. *Blekinge Institute of Technology Karlskrona, Sweden*

[11] Jean P. M., (2015), Fuzzy Eigen values and fuzzy Eigen Vectors. *ISPAC VOL* 2; 9- 11

[12] Pawlak, Z. (1982), Rough fuzzy sets, Algebraic and topological approach. *ICS PAS reports* (482); 12

[13] Zadeh (1965) Fuzzy sets. *Fuzzy sets information and control* (8); 338-353.

Corresponding author: Abubakar I.
ibrabu261@gmail.com
Department of Mathematics, Federal University of Education Zaria.